Math 2202: Multi-variable calculus **Instructor: Qile Chen**

11/13 - 11/16/2020 Midterm Exam 3 90 minutes

Name: Section:

Instructions.

(1) Submit your solutions on **Gradescope** under the course **MATH2202 01 & 02 - Exams and Quizzes** before 11:59 pm on 11/16.
(2) Once you have started the exam, you have **at most 90 minutes** to finish and submit your exam.
(3) Show *all* the steps of your work clearly. Little or no credit may be awarded, even when your answer is correct, if all work, including scratch work, is not shown.
(4) Indicate your final answers **clearly**.

Question	Points	Your Score
Q1	16	
Q2	10	
Q3	10	
Q4	14	
Q5	14	
≤ 90 mins	5	
TOTAL	69	

Question 1 [16 points] Answer the following questions. No justification is needed.

(1) Consider the following sets in $\mathbb{R}^n$:

$$
\begin{aligned}
A &= \{1, 2, 3\} \subset \mathbb{R}. \\
B &= (-\infty, 0] \cup [6, 7] \subset \mathbb{R}. \\
C &= \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 100\}. \\
D &= \{(0, y) \in \mathbb{R}^2 \mid y > 0\}. \\
E &= \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 10^{100}\}.
\end{aligned}
$$

(1a) Which sets are closed?

(1b) Which sets are bounded?

(2) Consider the function $f(x, y, z) = x \sin(y) \cos(z^2)$ and answer the following questions:

(2a) $\nabla f = (\qquad\qquad\qquad)$.

(2b) Find the **unit** vector along which the function $f(x, y, z)$ increases most rapidly at the point $(0, \frac{\pi}{2}, 0)$.

(3) Given $\nabla f(3, 0, 2) = (1, 1, 1)$, find the directional derivative of $f(x, y, z)$ at the point $(3, 0, 2)$ in the direction of $\mathbf{u} = (\frac{2}{3}, -\frac{2}{3}, \frac{1}{3})$.

(4) Let $f(x)$ be a continuous function defined on an interval $I \subset \mathbb{R}$.

(4a) Suppose $I = (0, 1)$. Can $f(x)$ always achieve its maximal and minimal values on I? If not, give a counterexample.

(4b) Suppose $I = [0, +\infty)$. Can $f(x)$ always achieve its maximal and minimal values on I? If not, give a counterexample.

Question 2 [10 points] Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?

Question 3 [10 points] Find all critical points of $f(x, y) = x^3 - 3xy + y^3$. Determine which critical points are local maxima, which are local minima, and which are saddle points.

Question 4 [14 points] Evaluate the following double integrals.

(1) $\iint_D y e^{xy} \, \mathrm{d}\, x \, \mathrm{d}\, y$, where $D = \{(x, y) \mid 0 \leq x \leq 2, \quad 0 \leq y \leq 1\}$.

(2) $\int_0^9 \int_{\sqrt{y}}^3 e^{x^3} \, \mathrm{d}\, x \, \mathrm{d}\, y$. (Hint: change the order of integration.)

Question 5 [14 points] Use the method of Lagrange multipliers to find the points on the curve $xy^2 = 16$ which are closest to the origin in the xy-plane.