Practice Final

Due Tuesday 12/14 9:00 a.m.

Everything is extra credit, so there is no possibility of you losing credit towards your total quiz grade for the course. In other words this is positively graded so the more correct work you show the more extra credit you get, while incorrect work is not penalized but just not counted towards the extra credit you will receive. The maximum amount of extra credit you can possibly receive is equivalent to that of 1.5 quizzes.

Time limit: None

Problem 1

Find the angle between the vectors $\langle 1, 1, 2\rangle$ and $\langle 13, 2, 11\rangle$.

Problem 2

Find the volume of the pralellopiped spanned by the vectors $\langle 1, 0, 0\rangle, \langle 1, 1, 0\rangle$, and $\langle 1, 1, 2\rangle$.

Problem 3

Given the points $P = (1, 1, -1), Q = (1, 2, 0)$, $R = (-2, 2, 2)$,

1. Find a vector perpendicular to both PQ and PR.
2. Find the equation of a plane passing through P, Q, and R.

Problem 4

Find the parametric equations of a line that passes through $(1, 1, 1)$ and is perpendicular to the unit sphere in $\mathbb{R}^3$.

Problem 5

Let L denote the line which passes through $(0, 0, 1)$ and is parallel to the line in the xy-plane given by $y = 2x$.

1. Give the parametric equations and symmetric equations of L.
2. Let $\mathcal{P}$ be the plane which passes through $(0, 0, 1)$ and is *perpendicular* to the line L. Give the equation of $\mathcal{P}$.
3. Suppose that the point $Q \neq (0, 0, 1)$ lies on L. Write down the method or formula you would use to find the point Q^* which is (all of the following must hold)
   (a) on L;
   (b) the same distance away from the point $(0, 0, 1)$ as Q;
   (c) and is on the *other* side of $\mathcal{P}$ from Q.

Problem 6

The position vector of a point P on a curve C is $r(t) = \langle 3\cos t, 5\sin t, 4\cos t\rangle$.

1. Show that the speed at the point is constant.
2. At what point $A = (a, b, c)$ does the curve pass through the yz-plane?

Problem 7

Let C be the curve on the xy-plane defined by the parabola $y = x^2$. Compute the curvature of C at every point on C.

Problem 8

Let

$$u = x^2 + yz, \quad x = pr\cos\theta, \quad y = pr\sin\theta, \quad z = p + r.$$

Find $\frac{\partial u}{\partial p}, \frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}$ when $p = 2, r = 3, \theta = 0$.

Problem 9

Let $F(x, y) = x^2y - xy^3$, and $P = (2, 1)$. Find the directional derivative of F at P in the direction $\langle \frac{3}{5}, \frac{4}{5}\rangle$.

Problem 10

Find the point on the plane $2x + y - z = 6$ which is closest to the origin using Lagrange multipliers.

Problem 11

1. Write down the change of coordinate formulas for spherical coordinates, i.e. write x, y, and z in terms of the spherical coordinates.
2. Using spherical coordinates, describe the region in $\mathbb{R}^3$ in the first octant that is bounded by the sphere centered at the origin with radius 4 .

Problem 12

Consider the following domains in $\mathbb{R}^2$:

$$D_1 = \{(x, y) \mid y = x^2\}$$

$$D_2 = \{(x, y) \mid 1 \le x^2 + y^2 < 2\}$$

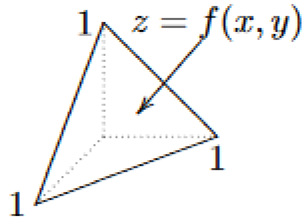
Which domains above is/are simply connected?
Which domains above is/are closed?
Which domains above is/are bounded?

Problem 13

Find the Jacobian corresponding to the change-of-variables from Cartesian coordinates to polar coordinates on the plane.

Problem 14

Find the volume of the tetrahedron shown below.



Tetrahedron

Problem 15

Sketch the region of integration and change the order of integration.

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y) \mathrm{d}y \mathrm{d}x$$

Problem 16

Find the volume of the solid bounded by the following: inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

Problem 17

Evaluate

$$\int_C (y - x)\mathrm{d}x + e^x \mathrm{d}y$$

where C is the line segment from $P = (1, 1)$ to $Q = (2, 4)$.

Problem 18

Evaluate the line integral

$$\int \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

where

$$\mathbf{F}(x, y, z) = \sin x\mathbf{i} + \cos y\mathbf{j} + xz\mathbf{k}$$

and

$$\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}, \quad 0 \leq t \leq 1.$$

Problem 19

Define what a conservative vector field is.

Problem 20

Let

$$\mathbf{F} = ay^2\mathbf{i} + 2y(x+z)\mathbf{j} + (by^2 + z^2)\mathbf{k}$$

be a vector field.

1. For what values of a and b will $\mathbf{F}$ be conservative?
2. Using these values, find a function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.
3. Using these values, give the equation of a surface S having the property

   $$\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r} = 0$$

   for a curve C from the point P to Q on the surface.

Problem 21

Evaluate the following integral directly, then evaluate it using Green's Theorem:

$$\oint_C (x-y)\mathrm{d}x + (x+y)\mathrm{d}y$$

where C is the circle centered at the origin with radius 2.

Problem 22

Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where

$$F(x, y) = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$$

and where C consists of the arc of the curve $y = \sin x$ from $(0, 0)$ to $(\pi, 0)$ and the line segment from $(\pi, 0)$ to $(0, 0)$.

Problem 23

Compute $\int_C \mathbf{F}\mathrm{d}\mathbf{r}$, where

$$\mathbf{F}(x, y) = \frac{2xy\mathbf{i} + (y^2 - x^2)\mathbf{j}}{(x^2 + y^2)^2}$$

and C is any positively oriented simple closed curve that encloses the origin.